April 3, 1996 WOCOMAL VARSITY MEET

ROUND I: Elementary Number Theory

ALL ANSWERS MUST BE EXPRESSED IN SIMPLEST EXACT FORM

1. An integer is divisible by 20, is a multiple of 30, and has 40 as a factor. What is the least positive integer that satisfies these conditions?

2. The numbers in this problem are all base 4. Find the sum of the least common multiple of $2_4$ and $12_4$, and the greatest common factor of $100_4$ and $30_4$. Give your answer in base 4.

3. Noting that 999999 = 1000000 – 1, factor 999999 into its prime factorization.

ANSWERS

( 1 pt ) 1._____

(2 pts) 2._____4_____

(3 pts) 3._____

Bancroft, Mass Academy, Tantasqua

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ROUND II: Algebra I — Open

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM.

1. Evaluate $\dfrac{-2^x}{2x^0}$ for $x = 2$.

2. Simplify: $$\frac{\dfrac{x+1}{x-1} + \dfrac{x-1}{x+1}}{\dfrac{x+1}{x-1} - \dfrac{x-1}{x+1}}$$

3. A five digit number is such that with a 2 after it, it is three times as large as it is with a 2 before it. (This compares two six- digit numbers.) What is the five digit number?

ANSWERS

( 1 pt ) 1._____

(2 pts) 2._____

(3 pts) 3._____

Auburn, Clinton, Shrewsbury, QSC

ROUND III: Theory of polynomial equations and functions, complex numbers

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. One of the zeros of $P(x) = x^3 + 4x^2 - 7x - 10$ is $-1$.
   Find the <u>other</u> zeros of P.

2. Evaluate $\left( \frac{1}{2} - \frac{i\sqrt{3}}{2} \right)^{23}$

3. $(x+1)(x+2)^2(x+3) - 72 = 0$ has two real and two imaginary roots. Find <u>either</u> imaginary root. Answer in simplest radical form with $i$, not decimal form, if a radical is involved.

ANSWERS

( 1 pt ) 1._____

(2 pts) 2._____

(3 pts) 3._____

Clinton, Quaboag, Shepherd Hill

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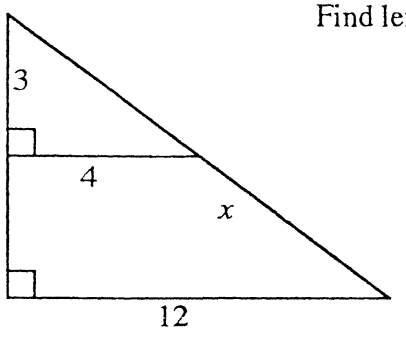
ROUND IV: Similarity and Pythagorean relationships

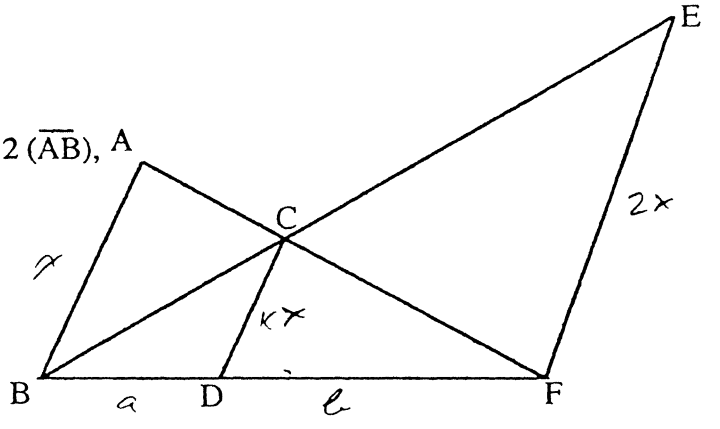
ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. Find length $x$




2. When the lower end of a ladder that had been resting flat against a wall was pulled out four feet, the top of the ladder descended one-fifth the length of the ladder. Find the length of the ladder.

3. If $\overline{AB} \parallel \overline{CD} \parallel \overline{EF}$ and $\overline{EF} = 2(\overline{AB})$, then $\overline{CD} = k(\overline{AB})$. Find k.




ANSWERS

( 1 pt ) 1._____

(2 pts) 2._____ feet _____

(3 pts) 3._____

Bromfield, Holy Name, Worcester Academy

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ROUND V: Trigonometry — open

ALL ANSWERS MUST BE IN SIMPEST EXACT FORM SPECIFIED IN THE PROBLEM

1. Steve and Joyce are observing the Washington Monument from one–quarter mile away. The monument is 555 feet tall. What is the angle of elevation from them to the top of the monument to the nearest degree? ( One mile = 5280 feet. The monument is on a slight rise, but assume that their eyes are at the level of the base of the monument. )

2. Find all degree values of $\theta$ between $0^\circ$ and $180^\circ$ inclusive for which
$\tan\theta \cdot \sin\theta + 1 = \tan\theta + \sin\theta$

3. For the function $f(x) = 1 + 4\sin(\frac{\pi}{3} - x)$, find, in terms of $\pi$, the smallest positive radian value $x$ that makes $f(x)$ a maximum.

ANSWERS

( 1 pt ) 1._____

(2 pts) 2._____

(3 pts) 3._____

Bartlett, Doherty, St. John's

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TEAM ROUND: Topics of previous rounds and open

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM, NO APPROXIMATIONS.

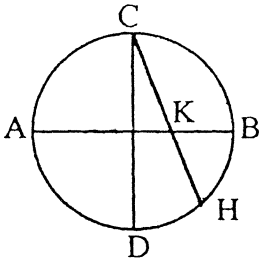
2 points each

1. A number is called "cute" if it has exactly 4 positive integer divisors. What percent of the first 25 positive integers are cute? Include itself and 1 as divisors.

2. If $$1+\cfrac{1+\cfrac{1+\cfrac{x}{1}}{1}}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{x}}}} = ax + b + \frac{r}{2x+1}$$, find $r$

3. Given that k and 2k are zeros of $f(x) = x^3 + 4x^2 + 9kx - 90$, find three zeros of $f(x)$ as specific numerical values.

4. In the circle at the right $\overline{AB}$ and $\overline{CD}$ are perpendicular diameters and the radius is 10. If $\overline{CH}$=16, find $\overline{BK}$.




5. Given $\sin 105^\circ = \frac{\sqrt{6}+\sqrt{2}}{4}$ and $\cos 105^\circ = \frac{\sqrt{2}-\sqrt{6}}{4}$

Find $\tan 52.5^\circ$ exactly in a form involving simplified radicals and no irrational denominatiors of fractions.

6. What is the smallest sum of the two largest angles, in degrees, in a scalene triangle in which all angle measures are whole numbers?

7. Solve: $(a^2 - 2)^2 = 3 + 2(a^2 - 2)$

8. If the diagonal of a rectangle is $4\sqrt{3}$ and its area is 8, find its perimeter.

9. What is the units digits in the sum $1! + 2! + 3! + 4! + \ldots + 25!$ ?

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ROUND I
# theory

1. 1 pt 120
2. 2 pts $22_4$
3. 3 pts $3^3 \cdot 7 \cdot 11 \cdot 13 \cdot 37$ any order

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ROUND II
Alg 1

1. 1 pt $-2$
2. 2 pts $\frac{x^2+1}{2x}$
3. 3 pts 85714

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ROUND III
poly eq
f(x)
complex

1. 1 pt $-5$ and $2$
2. 2 pts $\frac{1}{2} + \frac{i\sqrt{3}}{2}$
3. 3 pts Either $-2+2i\sqrt{2}$ or $-2-2i\sqrt{2}$

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ROUND IV
similar
Pythag

1. 1 pt 10
2. 2 pts $6\frac{2}{3}$ or $\frac{20}{3}$ or $6.\overline{6}$
3. 3 pts $\frac{2}{3}$ or $.\overline{6}$

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ROUND V
Trig

1. 1 pt $23^\circ$
2. 2 pts $45^\circ$
3. 3 pts $\frac{11\pi}{6}$

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TEAM ROUND 2 pts each

1. 28 %
2. $\frac{3}{4}$ or .75
3. $-6, -3, 5$ any order
4. $2\frac{1}{2}$ or $\frac{5}{2}$ or 2.5
5. $2 + \sqrt{6} - \sqrt{2} - \sqrt{3}$
6. $121^\circ$
7. $-1, 1, -\sqrt{5}, \sqrt{5}$ any order, $\pm$ OK
8. 16
9. 3